

SCOTCH
COLLEGE



Mathematics Department

Scotch College

Mathematics Specialist

Test One

Date: 4th December 2015

NAME: _____

Time allowed

Section	Reading	Working
Calculator-free	2 minutes	25 minutes
Calculator-assumed	2 minutes	25 minutes

Section One (Calculator-free): 27 marks

Permissible items:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet

Write your answers in the spaces provided.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use** pencil except in diagrams.

	Question	Marks available	Marks awarded
Calculator Free	1	6	
	2	6	
	3	10	
	4	5	
	Section One Total	27	
Calculator Assumed	5	7	
	6	4	
	7	7	
	8	7	
	Section Two Total	25	
	Total:	52	

Section One: Calculator-free**[27 marks]**

This section has **Four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 1 [6 marks]

Simplify each of the following expressions, writing your answer in exact polar form.

(a) $(\sqrt{3}-i)^2(\sqrt{3}-i)$

[2]

(b) $3cis\left(\frac{\pi}{4}\right) \times \left[2cis\left(\frac{-\pi}{3}\right)\right]^{-1}$

[2]

(c) $\frac{1}{\sqrt{2cis\left(\frac{\pi}{2}\right)}}$

[2]

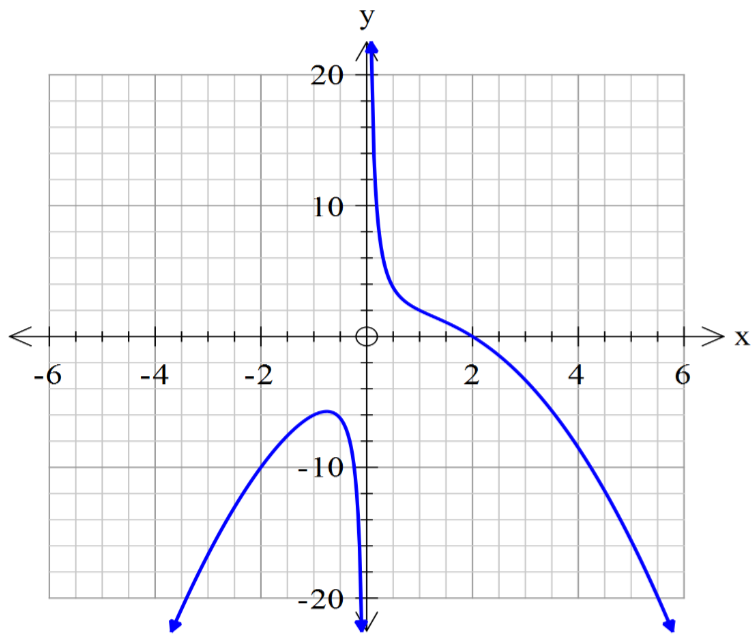
Question 2 [6 marks]

- (a) (i) Find the quotient and the remainder for $\frac{z^3 - 2z^2 + 4z - 1}{z^2 - z + 1}$, hence rewrite $z^3 - 2z^2 + 4z - 1$ in the form $H(z) \times (z^2 - z + 1) + R(z)$ [3]

- (ii) Hence, solve $z^3 - 2z^2 + 4z - 1 = 2z$ [3]

Question 3 [10 marks]

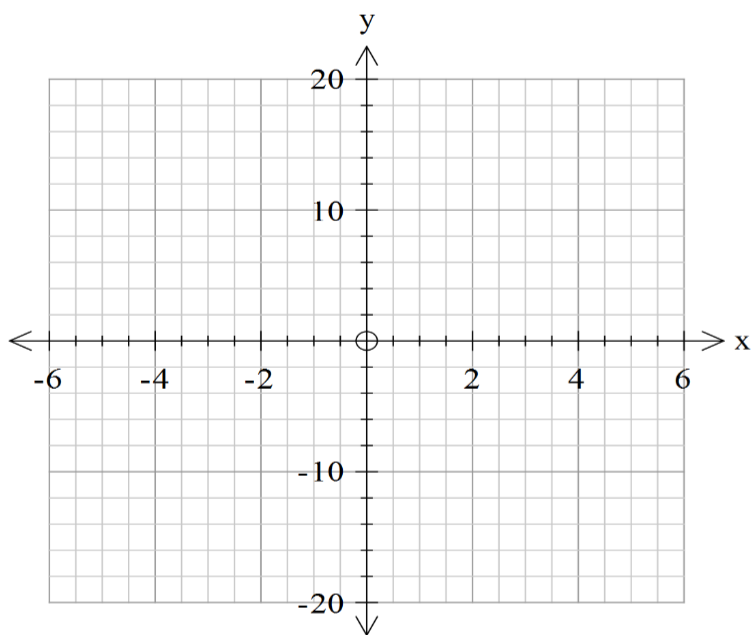
Given the graph of $y = f(x)$ is given as follows;



Sketch the graph of;

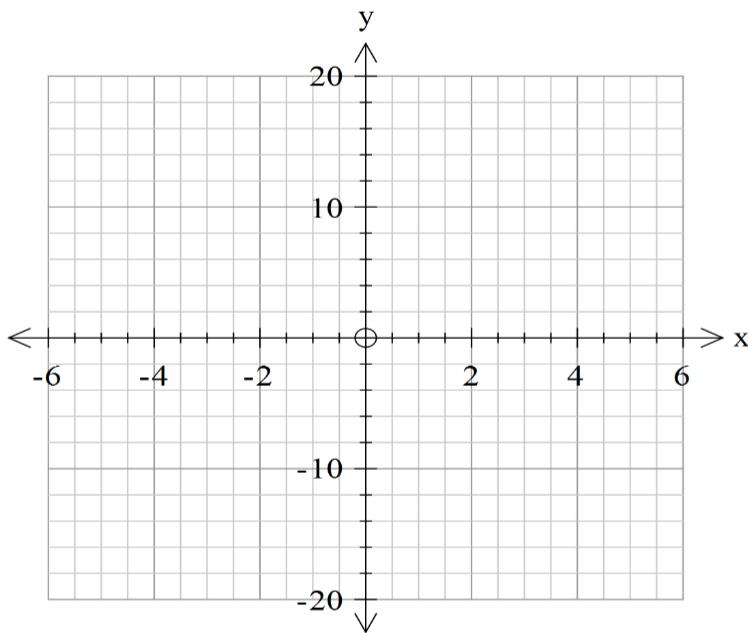
(a) (i) $y = f\left(\frac{x}{2}\right)$

[3]



(ii) $y = |f(x)| + 2.$

[3]



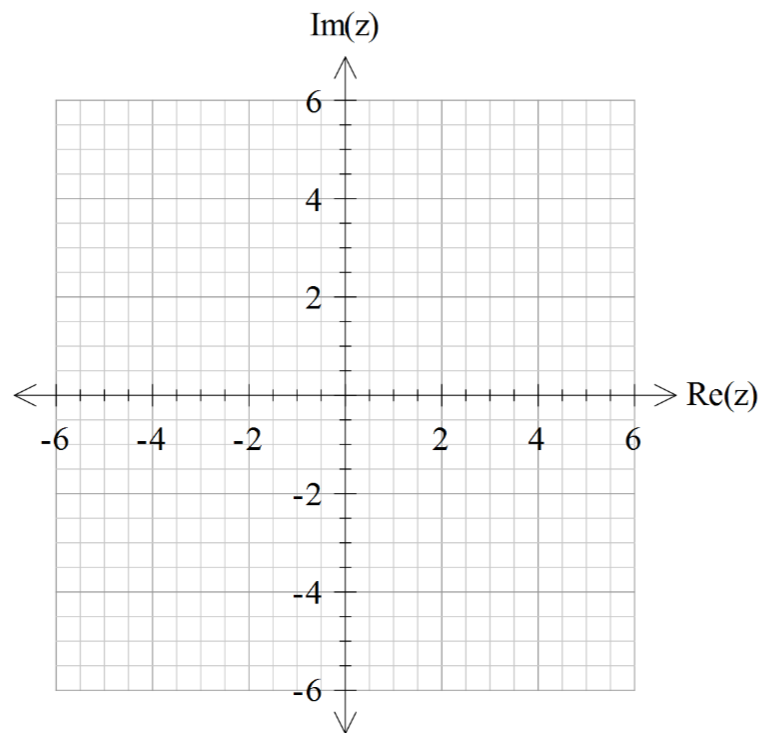
- (b) Given that $g(x) = \sqrt{3x-1}$ and $h(x) = \frac{x+2}{x+1}$, find the domain and range of the composite function $g \circ h(x)$

[4]

Question 4 [5 marks]

- (a) On an Argand diagram sketch the loci of points and that satisfy the following condition;

$$|z - 2i| \leq |z - 3 + i|$$



- (b) Give the equation of the locus in Cartesian form.

[3]

[2]

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Section	Reading	Working
Calculator-assumed	2 minutes	25 minutes

Section Two (Calculator-assumed): 25 marks

Permissible items:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators – CAS, graphic or scientific, satisfying the conditions set by the Curriculum Council for this course.

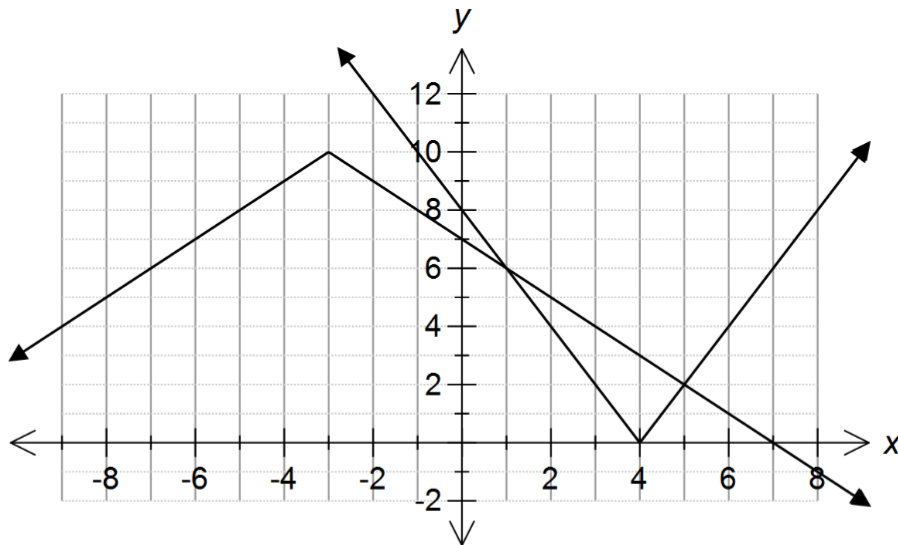
Write your answers in the spaces provided.

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Section Two: Calculator-assumed**[25 marks]**

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 5 [7 marks]

(a) Use the diagram above to solve for x in the following.

(i) $-|x + 3| + 10 = 7$

[1]

(ii) $-|x + 3| + 10 \geq |2x - 8|$

[2]

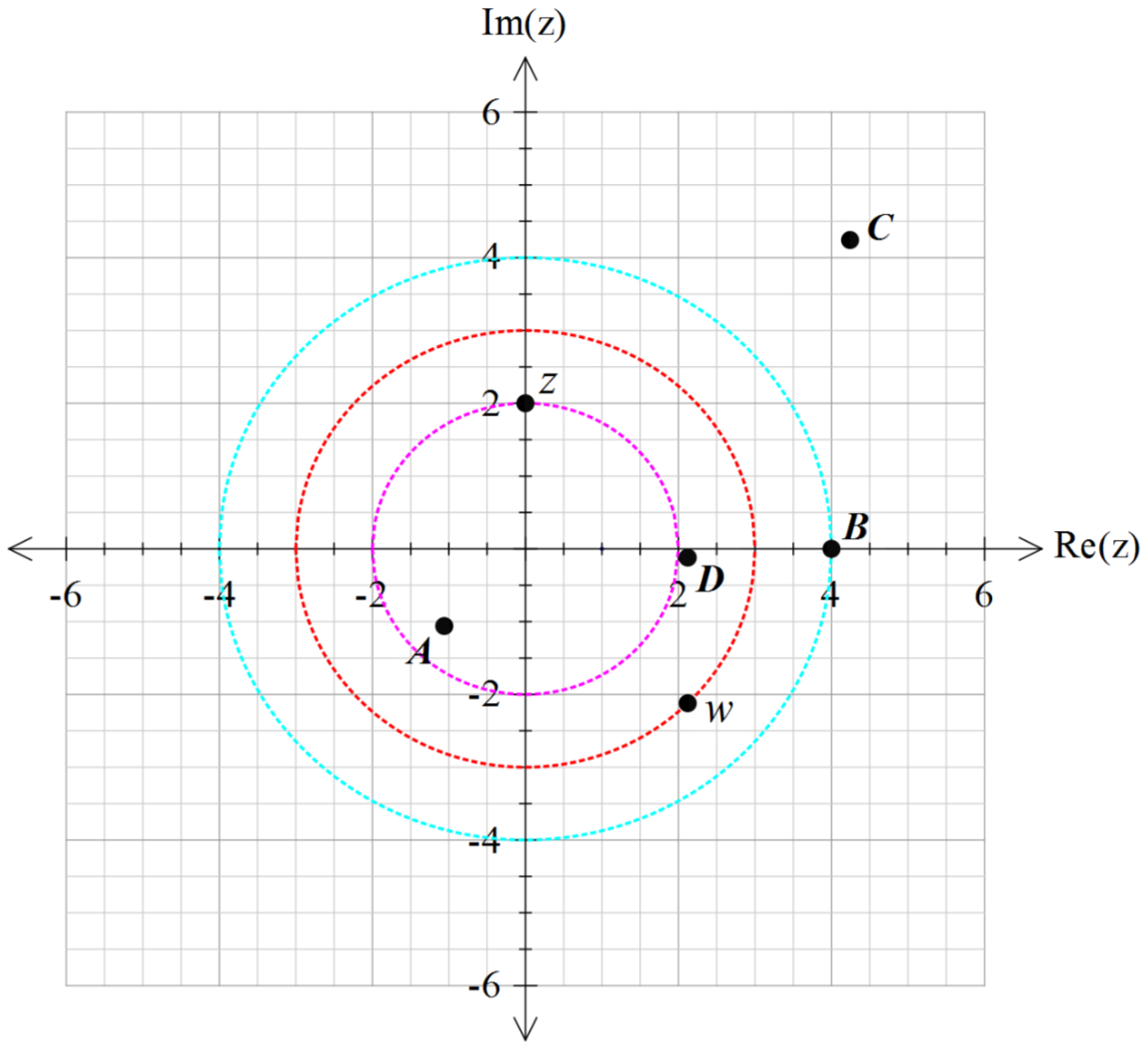
(b) Solve the following algebraically $4 + |3 + 2x| > |x - 5|$

[4]

Question 6 [4 marks]

Given the position of z and w on the Argand diagram below. Label the points A, B, C and D using the following options.

- $w + z$ wz $\frac{-1}{2}w$ $z\bar{z}$ $\frac{w}{z}$ w^{-2} z^2



A _____

[1]

B _____

[1]

C _____

[1]

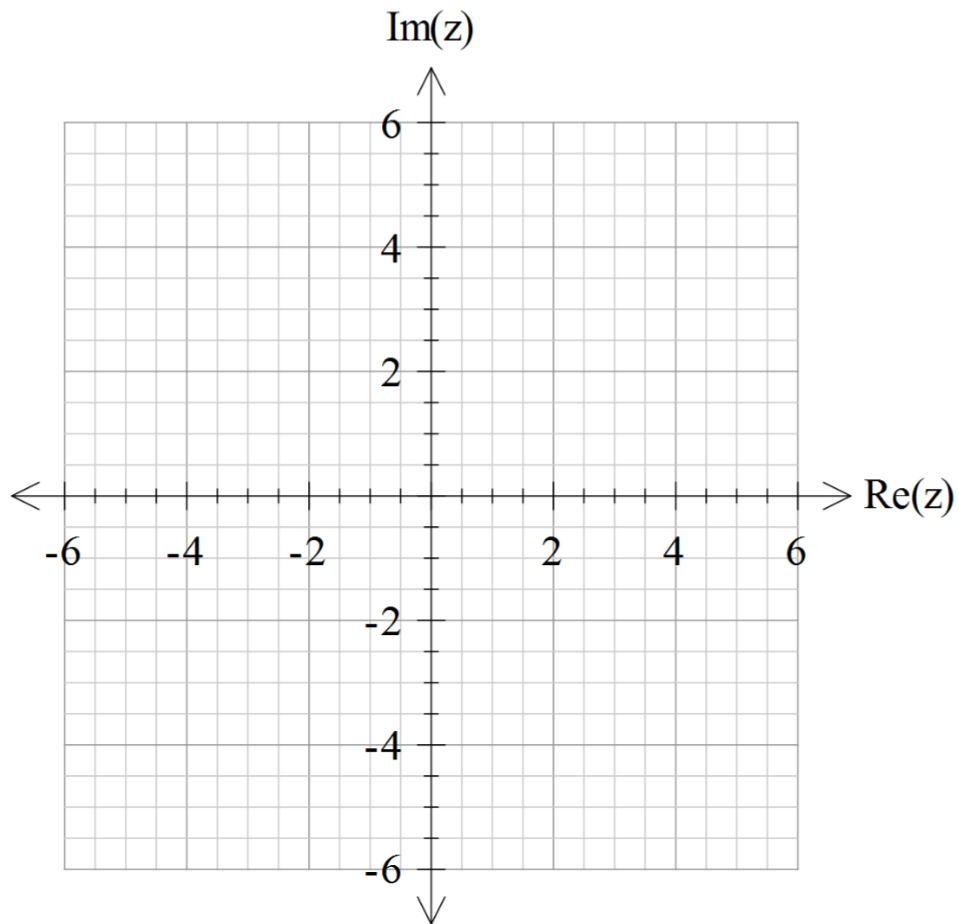
D _____

[1]

Question 7 [7 marks]

- (a) Represent on the Argand diagram provided below, the loci of points, that satisfy the following conditions;

$$|z + 2 - i| \leq 4 \quad , \quad -\frac{5\pi}{6} \leq \arg(z) < \frac{\pi}{3} \quad \text{and} \quad 4\operatorname{Im}(z) + 3\operatorname{Re}(z) + 8 \geq 0$$



[4]

- (b) Given that $|z + 2 - i| \leq 4$, state the minimum and maximum value of $|z|$.

[3]

Question 8 [7 marks]

- (a) Using your CAS calculator (or otherwise) find all the solutions to $z^5 = 512(\sqrt{3} - i)$ in exact polar form, where $z = r(\cos \theta + i \sin \theta)$, $-\pi < \theta \leq \pi$ and $r \geq 0$.

[4]

- (b) Draw the solutions from (a) on the complex plane below. Show all major features.

[3]

